ISSN 2229-5518

COUPLED GROUNDWATER SIMULATION-OPTIMIZATION MODELLING OF AGRARIAN SUB-CATCHMENT

A.O. Ibeje

Abstract—The numerical groundwater flow model, MODFLOW, was used to solve the 3-D groundwater flow equations and the results were input for the optimization of groundwater withdrawal. The groundwater model was applied at Awarra community in Ohaji-Egbema L.G.A. of Imo State. The unconfined aquifer was modelled with a finite-difference discretization of 200 rows and 100 columns. Eighteen out of 24 wells served as observation wells. The sensitivity analysis indicated that the natural processes have the most impacts on the groundwater budget of the aquifer. Using linear programming, the results of the optimization of groundwater withdrawal showed that rainfall recharge in the rainy season maintained the water levels within the constraints of drawdown. The results showed that coupling flow-simulation and optimization models could be a very useful procedure when solving groundwater management problems.

Index Terms—groundwater withdrawal; simulation; optimization; flow

1 Introduction

Groundwater reserves are being increasingly exploited all over the world for meeting irrigation, industrial and municipal demands (Vermeulen et al., 2010). Managing groundwater has two major components. The first component is simulation of groundwater flow which involves modelling the groundwater flow through porous media along with its linkages with other hydrologic systems (Middlemis, 2010).

The second management component is optimization of groundwater extraction which involves the determination of the maximum flow rate that can be pumped from a certain number of wells, subject to a given set of constraints limiting the drawdown values (Yazieigil and Rasheeduddin, 2015). Groundwater simulation model aids understanding of the behavior of the aquifer system whereas optimization of groundwater helps to mitigate the environmental and ecological side effects caused by over-exploitation of water resources (Delleur, 2010). This study was focused on coupled application of simulation and optimization in groundwater management.

2 Materials and Method

The study considered mean values of monthly rainfall, temperature and pan evaporation data over a period of 30 years (1980-2014) as made available by NIMET (2016) in Table 1. Data of aquifer parameter were based on field test conducted by SPDC (2010). Initial hydraulic heads for the model were sourced from the steady-state model completed by Nwofor (2014) in Table 2. The average pumping rates of wells were obtained from IWWDA (2016) as shown in Table 2.

- *Ibeje A.O. is currently a senior lecturer in the department of Civil Engineering at the Imo State University, Owerri, Nigeria, PH-+2348039381505. E-mail: engineeribeje@yahoo.com*

2.1 Study Area

Awarra is located at 5.350 latitude and 6.76670 longititude on the southern part of Ohaji/Egbema L.G.A (Okorie et al., 2012). Ohaji-Egbema Local Government Area lies in the southwestern part of Imo state, Nigeria. The lithologs showed a multi-aquifer system with the top unconfined aquifer zone varied in thickness from 30-40m (Odoemene, 2013). Increase in population due to agricultural production and crude oil exploration has resulted in the average drawdown of 4.14 x 10 m year-1 (SPDC, 2012). The intense agricultural activities in the area have increased water demand for irrigation (Amadi et al., 2010). There has been a decrease in water replenishment to groundwater, so farmers over pumped groundwater for irrigation. The area was therefore chosen for the application of the coupled groundwater simulation-optimization model.

2.2 Simulation Model

The 3-D groundwater flow is:

$$\frac{\partial}{\partial x}\left[T_x\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[T_y\frac{\partial h}{\partial y}\right]+\frac{\partial}{\partial z}\left[T_z\frac{\partial h}{\partial z}\right]=-R^1+P+S\frac{\partial h}{\partial t} \quad (1)$$

R^1 = Rate of water produced (rainfall), P = Rate of water consumed (pumping well and evapotransportation), s = the specific storage of the porous material. Under steady-state conditions, the flow through isotropic medium is given by

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0 \quad (2)$$

The groundwater flow equation was simplified by assuming an aquifer of homogeneous, isotropic sand mass in which the flow was laminar. The aquifer recharge rate of 90% of the average annual rainfall was also assumed (Hills, 2010). Evapotranspiration losses from vegetation were assumed as 10% of the pan evaporation (Domenico, 2010). Drichlet, Neumann

ISSN 2229-5518

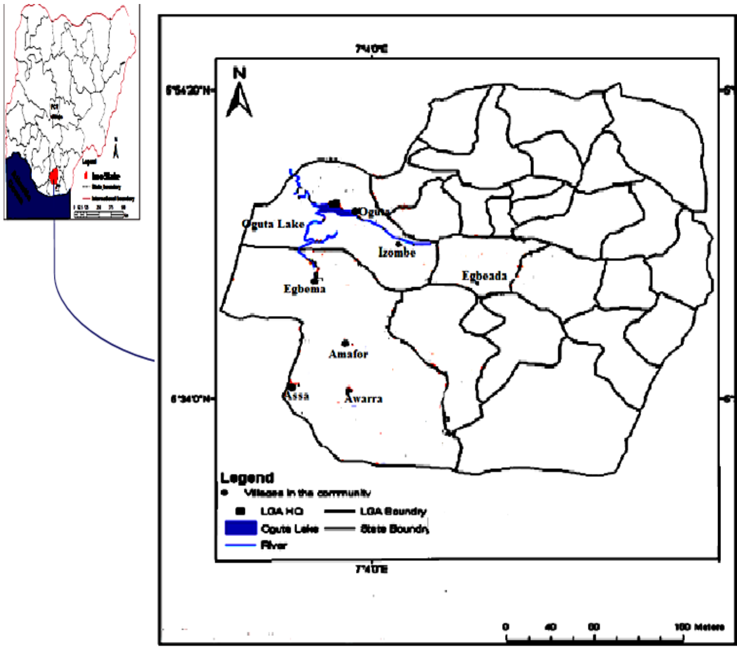

Figure 1: Location of the Study Area

south boundaries of the model domain. Head data was received from 14 observation wells. Using the MODFLOW software, a grid of 200 rows and 100 columns that covered an area of 71 km² represented the aquifer. The location of the wells in the aquifer is indicated in Figure 2. The model calibration was done using automatic model calibration interface called PEST in MODFLOW (Zeng, 2013). The model was validated with the input data from November, 2014 condition; compiled by Nwofor (2014). Some parameters, e.g. K-values, were adjusted to fit November, 2014 observed heads data.

2.3 Optimization Model

The objective of the optimization model was to maximize the total pumping rate from a certain the constraints limiting the well drawdown and the irrigation water demand of the area. The linear programming model was formulated as:

$$Max \sum_J QT_j{}^k \tag{3}$$

Subject to the linear constraints

$$s_j{}^k = (H_0)_j - H_j = \sum_J \alpha_{ij} QT_J \tag{4}$$

$$\sum_j Q_j{}^k \geq D^k \tag{5}$$

$$QT_j, Q_{JK} \geq 0 \tag{6}$$

where QT_j^k= the total pumping rate at well j (m³/s) at month k; s_j^k= drawdown at well j in month k (m); H_J= hydraulic head at well j (m); Q_j^k= pumping rate of well j for the month k (m³/s); α_{ij} = aquifer influence coefficient describing the change of head at node i with respect to a change in pumping rate at well j; H_{Oj}= initial hydraulic head at well j (m)

3 Results and Discussion

Table 1 shows the descriptive statistics of the rainfall and evaporation data used inputted in model. Table 2 shows the initial hydraulic heads and mean monthly pumping rates of the wells used for the simulation.

3.1 Results of Steady-State Simulation of Groundwater Flow

The simulated heads H_j^k for each month, k and each well j, are shown in Table 3.The results of MODFLOW simulation in Table 5 indicated that the water level of the aquifer takes a maximum value of 18.7m in well W_{13} in September.

When compared to the initial head of 1.7m in the same well in April, this value appears to be very high. This is because the month of September has the highest monthly rainfall, which in turn recharges the aquifer.

The minimum value (0.58m) of water level in the aquifer occurred in April, March, October and November, during the dry season at Well W2. The drawdown in this case was quite obvious, compared to the initial head of 5.5m (Table 2). Hence, these months are considered to be the most critical. It should be noted that among these months, the month of November had one of the lowest record of groundwater recharge from rainfall (58mm). This follows from the findings of Ibeje et al. (2012) who had similar results when they optimized water resources in semi-arid region of Nigeria. The comparisons between computed groundwater head and the observed heads from November, 2014 conditions showed a very good match with a correlation coefficient of 0.99 (Figure 2). In this study, the sensitivity analysis was performed by varying (increasing or decreasing) by 10 %, the input parameters including K-value, recharge, pumping rate and heads. The results of the sensitivity analyses are summarized in Table 4.

The sensitivity analysis shows that recharge and K-values cause the greatest change in the simulation heads with the heads 6.2 to 8.9 % different from the normal condition (Table 4). The other parameters show small differences and less sensitive to the changes. This is consistent with the findings of Anderson (2013). On a spatial scale, wells W1, W3 to W8 were located near the riverine communities in neighboring Rivers state. As such, they recorded optimal flow rates throughout all the months of the year. This was unlike wells W9 to W14 which were located near communities in Imo state. These communities in Imo state have lower water table than the communities in Rivers state (Ehirim and Nwwankwo, 2010).

3.2 Results of Optimization of Groundwater Extraction

From Equation 5, the water demand constraints, D^k, of Awarra aquifer for each month, i.e from January (k=1) to December (k=12) at well j (j=1.. .18), is as follows:

$$\sum_{j=1}^{18} Q_j^1 = Q_{w1}^1 + Q_{w2}^1 + \cdots + Q_{w18}^1 \geq -21464m^3day^{-1} \quad (7)$$
$$\sum_{j=1}^{18} Q_j^2 = Q_{w1}^2 + Q_{w2}^2 + \cdots + Q_{w18}^2 \geq -20464m^3day^{-1} \quad (8)$$
$$\sum_{j=1}^{18} Q_j^3 = Q_{w1}^3 + Q_{w2}^3 + \cdots + Q_{w18}^3 \geq -27464m^3day^{-1} \quad (9)$$
$$\sum_{j=1}^{18} Q_j^4 = Q_{w1}^4 + Q_{w2}^4 + \cdots + Q_{w18}^4 \geq -26464m^3day^{-1} \quad (10)$$
$$\sum_{j=1}^{18} Q_j^5 = Q_{w1}^5 + Q_{w2}^5 + \cdots + Q_{w18}^5 \geq -48136m^3day^{-1} \quad (11)$$
$$\sum_{j=1}^{18} Q_j^6 = Q_{w1}^6 + Q_{w2}^6 + \cdots + Q_{w18}^6 \geq -48136m^3day^{-1} \quad (12)$$
$$\sum_{j=1}^{18} Q_j^7 = Q_{w1}^7 + Q_{w2}^7 + \cdots + Q_{w18}^7 \geq -47136m^3day^{-1} \quad (13)$$
$$\sum_{j=1}^{18} Q_j^8 = Q_{w1}^8 + Q_{w2}^8 + \cdots + Q_{w18}^8 \geq -48136m^3day^{-1} \quad (14)$$
$$\sum_{j=1}^{18} Q_j^9 = Q_{w1}^9 + Q_{w2}^9 + \cdots + Q_{w18}^9 \geq -46136m^3day^{-1} \quad (15)$$
$$\sum_{j=1}^{18} Q_j^{10} = Q_{w1}^{10} + Q_{w2}^{10} + \cdots + Q_{w18}^{10} \geq -22464m^3day^{-1} \quad (16)$$
$$\sum_{j=1}^{18} Q_j^{11} = Q_{w1}^{11} + Q_{w2}^{11} + \cdots + Q_{w18}^{11} \geq -23464m^3day^{-1} \quad (17)$$
$$\sum_{j=1}^{18} Q_j^{12} = Q_{w1}^{12} + Q_{w2}^{12} + \cdots + Q_{w18}^{12} \geq -20464m^3day^{-1} \quad (18)$$

Table 1: Descriptive Statistics of Monthly Evaporation and Rainfall (mm) Data

Month	Jan	Feb	Mar	Apr	may	Jun	Jul	Aug	sep	Oct	Nov	Dec	
Pumping Rate (m^3/day)	21464	20464	27464	26464	48136	48136	47136	48136	46136	22464	23464	20464	
Well	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}	W_{13}	W_{13}
Initial Head (m)	7.5	5.5	6.5	7.9	8.8	4.5	5.7	2.7	3.7	5.6	6.3	6.2	1.7

Table 2: Pumping Rate and Initial Hydraulic Heads Data

Statistics	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Mean Evaporation	**5.29**	**26.77**	**46.09**	**96.73**	**138.72**	**210.68**	**120.35**	**113.80**	**222.35**	**196.71**	**7.79**	**0.81**
Standard Deviation of Evaporation	11.39	27.49	26.33	51.20	75.80	59.16	55.16	85.56	64.88	95.22	13.60	1.96
Mean Rainfall	**21.30**	**38.55**	**112.15**	**176.01**	**256.56**	**329.08**	**326.43**	**333.90**	**384.73**	**267.01**	**58.05**	**15.93**
Standard Deviation of Rainfall	27.64	40.10	58.67	69.85	99.01	113.24	131.15	126.37	126.00	97.01	43.35	24.30
Sample Size	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00

Table 3: Simulated Heads for all Time Periods in all wells (m)

Wells	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
W_1	6.72	6.72	6.72	6.72	6.72	6.72	6.72	6.72	6.72	6.72	6.72	6.72
W_2	5.5	5.5	**0.58**	**0.58**	3.18	3.18	3.18	3.18	3.18	**0.58**	**0.58**	**0.58**
W_3	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28
W_4	3.95	1.35	5.76	4.76	8.59	8.59	8.59	8.59	2	0.75	1.73	1.45
W_5	8.71	7.25	8.71	8.71	7.25	7.25	7.25*	7.25	7.25	8.71	8.71	8.71
W_6	8.87	8.87	8.87	8.87	14.6	14.6	14.6*	14.6	14.6	8.87	8.87	8.87
W_7	5.56	5.56	5.56	5.56	1.64	1.64	1.64*	1.64	1.64	5.56	5.56	5.56
W_8	2.41	2.2	2.41	2.41	2.41	2.41	2.41*	2.41	2.41	2.41	2.41	1.92
W_9	3.7	3.7	3.7	3.7	3.19	3.19	3.19	3.19	3.19	3.7	3.7	3.7
W_{10}	5.46	5.6	5.46	5.46	5.58	5.58	5.58	4.16	5.36	5.46	5.46	5.46
W_{11}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
W_{12}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
W_{13}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
W_{14}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

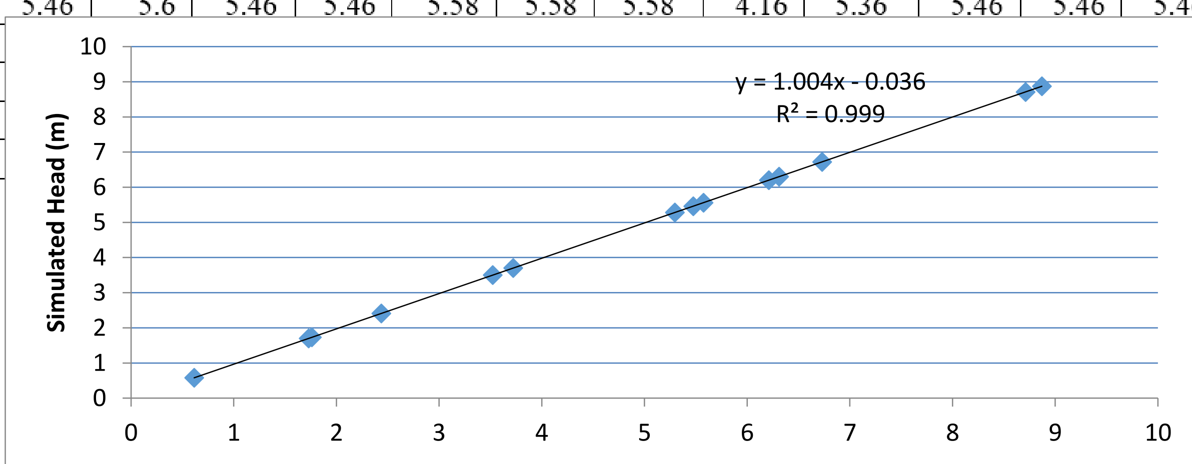

Figure 2: computed and observed groundwater head for November, 2014

ISSN 2229-5518

Thus by this finding, this result confirms similar reports by Ibe and Onu (2009). On a temporal scale, optimal flow rates were recorded in the months of May through September which are the rainy months in the area. The highest flow rate of 16370m^3/day was recorded in W4 between May and September.

But this value dropped to 64301m^3/day in December owing to high evaporation and lack of rainfall to recharge the aquifer; a feature characteristic of dry season (NIMET, 2016). As described in the study area, the month of January, February and March was the main irrigation period because of the commencement of the first planting season. It was also characterized with absence of rainfall, drop in water table due to low recharge and over pumping of aquifers. This translated to increased demand on the groundwater as the only reliable source of water. The model produced a total optimized value of 21464, 20464 and 48135m^3/day in the respective months. In the months of May, June, July August and September, the frequency of the rain had improved remarkably leading to reduced demand on water demand as users have alternative source of water such as rainfall harvesting etc. Therefore, for these months, there were optimum flow rates in all the wells under study. Wells W16 to W18 could not record optimum

Table 4 Results of Sensitivity Analysis with Change in Hydraulic Heads

Parameter input	Well	Normal condition		10% increase		10% decrease		% diff. from normal	
		computed	Residual	Computed	Residual	Computed	Residual	10% increase	10% decrease
Recharge	W_5	7.25	0.05	8.09	-0.50	6.98	0.61	7.2	-7.5
	W_6	14.6	-0.01	13.54	-0.90	11.72	0.92	7.0	-7.3
	W_7	1.64	-0.09	9.20	-0.74	7.86	0.60	7.7	-8.1
	W_8	2.4	-0.05	3.96	-0.37	3.32	0.27	8.8	-8.9
k-value	W_5	7.25	0.05	7.07	0.52	8.11	-0.52	-6.3	7.5
	W_6	14.6	-0.01	11.87	0.77	13.56	-0.92	-6.2	7.2
	W_7	1.64	-0.09	7.95	0.51	9.24	-0.78	-7.0	8.1
	W_8	2.4	-0.05	3.37	0.22	3.97	-0.38	-7.4	8.9

The drawdown constraints, s_i^k, for each month, k and each well, j are shown in Table 5.

Table 5: Minimum Well j Drawdown (s_i^k) for all months (m)

Wells	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
W_1	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78
W_2	0.00	0.00	6.08	6.08	8.68	8.68	8.68	8.68	8.68	6.08	6.08	6.08
W_3	1.22	1.22	1.22	1.22	1.22	1.22	1.22	1.22	1.22	1.22	1.22	1.22
W_4	11.85	9.25	13.66	12.66	16.49	16.49	16.49	16.49	9.90	8.65	9.63	6.45
W_5	0.09	1.55	0.09	0.09	1.55	1.55	1.55	1.55	1.55	0.09	0.09	0.09
W_6	13.37	13.37	13.37	13.37	19.10	19.10	19.10	19.10	19.10	13.37	13.37	13.37
W_7	0.14	0.14	0.14	0.14	4.06	4.06	4.06	4.06	4.06	0.14	0.14	0.14

Table 6: Optimum Well (j) Flow Rates, $QT_j{}^k$ for all Time Period, k (m^3/day)

Wells	Jan	Feb	May	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
W_1	739	739	739	739	739	739	739	739	739	739	739	739
W_2	0	0	4200	4200	6000	6000	6000	6000	6000	4200	4200	4200
W_3	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
W_4	11765	9182	13565	12565	16370	16370	16370	16370	9825	8585	9565	6401
W_5	100	1715	100	100	1715	1715	1715	1715	1715	100	100	100
W_6	7560	7560	7560	7560	10800	10800	10800	10800	10800	7560	7560	7560
W_7	100	100	100	100	2905	2905	2905	2905	2905	100	100	99
W_8	100	168	100	100	100	100	100	100	100	100	100	264
W_9	0	0	0	0	238	238	238	238	238	0	0	0
W_{10}	100	0	100	100	7868	7868	7868	6868	168	100	100	100
W_{11}	0	0	0	0	100	100	100	100	3432	0	0	0
W_{12}	0	0	0	0	100	100	100	100	100	0	0	0
W_{13}	0	0	0	0	100	100	100	100	4224	0	0	0
W_{14}	0	0	0	0	100	100	100	100	2640	0	0	0

flow rates in all the months and were therefore removed from Table 6.

4 Conclusion

A steady-state groundwater flow model was developed to simulate the existing hydrological system and the dominant processes that control groundwater flow in Awara community. The numerical model was calibrated against existing data The results served as the input for the optimization of groundwater withdrawal in the area. The groundwater extraction from the wells was optimized subject to constraints limiting the drawdown values using linear programming. It is recommended that future studies should focus on: the spatial distribution of the K-values and model validation with observation data that covers a longer period. An accurate method for recharge estimation which will take into account the relationship of rain and temperature, and the flow in unsaturated and saturated zones, is recommended. It is recommended that good understanding of recharge process under different climatic scenarios and spatially distribution of recharge is also needed for a more accurate recharge estimation and consequently more accurate groundwater resources assessment and management.

References

[1] Delleur, J.W. (2010). The Handbook of Groundwater Engineering. 2nd Edn. Boca Raton: CRC Press.

[2] Domenico, P.A. (2012). Concepts and Models in Groundwater Hydrology, McGraw-Hill, N.Y., 405 p

[3] Domenico, P.A. (2012). Concepts and Models in Groundwater Hydrology, McGraw-Hill, N.Y., 405 p

[4] Ehirim, C.N. and Nwankwo, C.N. (2010): Evaluation of aquifer characteristics and groundwater quality using geoelectric method in Choba, Port Harcourt. Scholars research Library, 2(2): 396-403

[5] Hill, M.C. (2010). Preconditioned Conjugate-Gradient 2 (PCG2). A Computer program for solving groundwater flow equations, U.S. Geological Survey, Denver

[6] Ibeje, A.O., Agunwamba, J.C. & Okoro, B.C. 2010. Allocation of Water Resources in Semi-Arid Region of Northern Nigeria. NIJOTECH, Vol. 29(3): 12-2

[7] IWADA (2016). Annual Report on Groundwater Pumping Rates in Imo State.

[8] Middlemis, H. (2010). Groundwater flow modeling guideline: Murray-Darling Basin Commission Aquaterra Consulting Pty Ltd, www.mdbc.gov.au

[9] NIMET (2016). Climatic Conditions of Lagos, Nigeria. Nigeria Meteorological Agency (NIMET), Abuja, Nigeria.

[10] Nwofor, J.C. (2014). Environmental Impact Assessment for Sustainable Development: The Nigeria Perspective. Enugu: EDPCA

[11] Odoemene, A. (2013). Social consequences of environmental changes in Niger Delta of Nigeria, Journal of Sustainable Development, Vol. 4(2):123-135

[12] Okorie, F.C., Okeke, I.C., Nnaji, A.O., Chibo, C.N. and Pat-Mbano, E.C. (2012). Evidence of Climate Variability in Imo State of Southeastern Nigeria. BALWOIS 2012 – Ohirid, Republic of Macedonia – 28 May -2 June 2012

[13] SPDC (2010). Environmental Impact Assessment (EIA) of Egbema, Egbema West and Ugada Fields Integrated Oil and Gas Development Project. The Shell Petroleum Development Company Limited, Port Harcourt, Nigeria.

[14] Vermeulen, P.T.M., Stroet, C.B.M.T. and Heemink, A.W. (2010). Limitations to upscaling of groundwater flow models dominated by surface water interaction. Water Resources Research 42 (10), 310-324

[15] Yazieigil, H. and Rasheeduddin, M. (2015). Optimization model for groundwater management in multi-aquifer systems. J. Water Resour: Planning Manage. Div. Am. Soc. Civ. Eng. 113:257-273

[16] Zheng, C. (2013). MT3D Version DoD_1.5, a modular three-dimensional transport model. The Hydrogeology Group, University of Alabana